Take care of

Take care of means ‘look after’.

Who takes care of sick people?
If I give you a rabbit, will you be able to take care of it properly?
When used without a preposition, take care means ‘careful’.

Take care when you are crossing the road.

Care about

When you care about something, you feel it is important or it interests or worries you.

I don’t care about your opinion. (= I don’t think that your opinion is important.)
If there is no object, we do not use about.

‘I don’t want to meet you again.’ ‘I don’t care.’ (NOT I don’t care about.)
About is also left out before a conjunction.

I don’t care whether you like it or not. (NOT I don’t care about whether you like it or not.)
Couldn’t care less means ‘don’t care at all’.

‘She is very upset with you.’ ‘I couldn’t care less.’ (= I don’t care at all.)

Care for

In a formal or literary style, care for can be used to mean ‘look after’.

He spent years caring for his sick mother.
Care for can also mean ‘like’.

He really cares for his siblings. (= He is really fond of his siblings.)
I don’t much care for fish. (= I don’t like fish.)